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MINKOWSKI LACES

Keywords: *Minkowski point set operations, Minkowski combinations, lace curves, modeling curves, differential properties of lace curves.*

A family of curves named "Minkowski Laces" is introduced and investigated, while some of their intrinsic geometric properties are derived and presented. Particular representatives of this two-parametric family of curves can be generated by means of Minkowski summative or Minkowski multiplicative combinations of two equally parameterized curve segments in the Euclidean space $\mathbf{E}^n$. Let two curve segments be determined by respective vector maps defined on the same interval $I \subset \mathbf{R}$

$$\mathbf{rk}(u) = (xk_1(u), xk_2(u), \ldots, xk_n(u)) \tag{1}$$

$$\mathbf{rl}(u) = (xl_1(u), xl_2(u), \ldots, xl_n(u)) \tag{2}$$

Minkowski summative combination of curves K and L

$$S = aK \oplus bL \tag{3}$$

is a family of curve segments in $\mathbf{E}^n$ parametrically represented on $I \subset \mathbf{R}$ by vector maps

$$S: \mathbf{s}(u) = a.\mathbf{rk}(u) + b\mathbf{rl}(u) = (xs_1(u), xs_2(u), \ldots, xs_n(u)) \tag{4}$$

where

$$xs_i(u) = \mathrm{a}.xk_i(u) + \mathrm{b}.xl_i(u)), \qquad i = 1, 2, \ldots, n. \tag{5}$$

Differential characteristics of the two-parametric family of summative laces can be derived and represented by means of derivatives of vector representations of the two operand curves.

Minkowski multiplicative combination of curves K and L

$$P = aK \otimes bL \tag{6}$$

is a family of curve segments in $\mathbf{E}^d$, $d = n(n-1)/2$, represented on $I \subset \mathbf{R}$ by vector maps

$$P: \mathbf{p}(u) = a.\mathbf{rk}(u) \wedge b\mathbf{rl}(u) = (xp_1(u), xp_2(u), \ldots, xp_d(u)) \tag{7}$$

where the following relations hold for coordinate functions

$$xp_k(u) = a.b.(xk_i(u)xl_j(u) - (xk_j(u)xl_i(u)), \qquad i, j = 1, 2, \ldots, n, k = 1, 2, \ldots, d. \tag{8}$$

Differential characteristics of the two-parametric family of multiplicative laces can be also derived from derivatives of vector maps of the two respective curves.

Illustrations of both Minkowski summative and multiplicative combinations are presented in following figures. Curves are well adjustable with respect to their forms and shapes, therefore they can be used in geometric modeling of shapes in computer graphics algorithms.

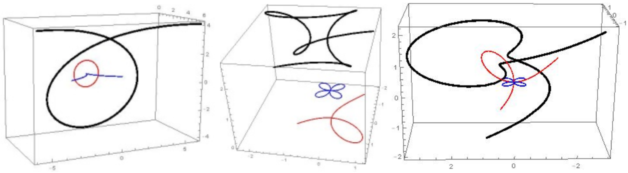

Fig. 1 Minkovski summative combinations of two curve segments.

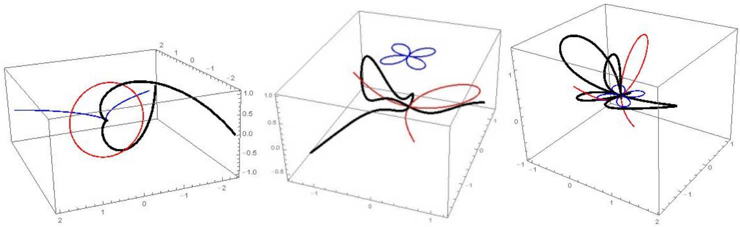

Fig. 2 Minkovski multiplicative combinations of two curve segments.

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